September 2010

MADALGO seminar by Pooya Davoodi, Aarhus University

On Space Efficient Two Dimensional Range Minimum Data Structures

Abstract:

The two dimensional range minimum query problem is to preprocess a static two dimensional m by n array A of size $N=m\cdot n$, such that subsequent queries, asking for the position of the minimum element in a rectangular range within A, can be answered efficiently.

We study the trade-off between the space and query time of the problem. We show that every algorithm enabled to access A during the query and using $O(N/c)$ bits additional space requires $\Omega(c)$ query time, for any c where $\{1 \leq c \leq N\}$. This lower bound holds for any dimension. In particular, for the one dimensional version of the problem, the lower bound is tight up to a constant factor. In two dimensions, we complement the lower bound with an indexing data structure of size $O(N/c)$ bits additional space which can be preprocessed in $O(N)$ time and achieves $O(c\cdot log^2\ c)$ query time. For $c=O(1)$, this is the first $O(1)$ query time algorithm using optimal $O(N)$ bits additional space. For the case where queries cannot probe A, we give a data structure of size $O(N\cdot\min\{m,log\ n\})$ bits with $O(1)$ query time, assuming $m\leq n$. This leaves a gap to the lower bound of $\Omega(N\cdot\log\ m)$ bits for this version of the problem.

Joint work with Gerth Stølting Brodal and Srinivasa S. Rao