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MADALGO seminar by Mohammad Ali Abam, Aarhus University

## Geometric Spanners for Weighted Point Sets

### Abstract:

Let  $(S, d)$  be a finite metric space, where each element  $p \in S$  has a non-negative weight  $wt(p)$ . We study  $t$ -spanners for the set  $S$  with respect to the following weighted distance function  $d\omega$ :

$$\begin{aligned}d\omega(p, q) &= 0 \quad \text{if } p = q, \\d\omega(p, q) &= wt(p) + d(p, q) + wt(q) \quad \text{if } p \neq q.\end{aligned}$$

We present a general method for turning spanners with respect to the  $d$ -metric into spanners with respect to the  $d\omega$ -metric. For any given  $\varepsilon > 0$ , we can apply our method to obtain  $(5 + \varepsilon)$ -spanners with a linear number of edges for three cases: points in Euclidean space  $\mathbb{R}^d$ , points in spaces of bounded doubling dimension, and points on the boundary of a convex body in  $\mathbb{R}^d$  where  $d$  is the geodesic distance function.

We also describe an alternative method that leads to  $(2 + \varepsilon)$ -spanners for points in  $\mathbb{R}^d$  and for points on the boundary of a convex body in  $\mathbb{R}^d$ . The number of edges in these spanners is  $O(n \log n)$ . This bound on the stretch factor is nearly optimal: in any finite metric space and for any  $\varepsilon > 0$ , it is possible to assign weights to the elements such that any non-complete graph has stretch factor larger than  $2 - \varepsilon$ .

Joint work with:

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