

Maj 2012

MADALGO seminars by Kasper Green Larsen, Aarhus University

### The Cell Probe Complexity of Dynamic Range Counting

#### Abstract:

In this talk we present a new technique for proving lower bounds on the update time and query time of dynamic data structures in the cell probe model. With this technique, we prove the highest lower bound to date for any explicit problem, namely a lower bound of  $t_q = \Omega(\frac{n}{w} \lg(w t_u))^2$ . Here  $n$  is the number of update operations,  $w$  the cell size,  $t_q$  the query time and  $t_u$  the update time. In the most natural setting of cell size  $w = \Theta(\lg n)$ , this gives a lower bound of  $t_q = \Omega(\frac{n}{\lg n} \lg \lg n)^2$  for any polylogarithmic update time. This bound is almost a quadratic improvement over the highest previous lower bound of  $\Omega(\lg n)$ , due to P{v a}tra{c s}cu and Demaine [SICOMP'06].

We prove our lower bound for the fundamental problem of weighted orthogonal range counting. In this problem, we are to support insertions of two-dimensional points, each assigned a  $\Theta(\lg n)$ -bit integer weight. A query to this problem is specified by a point  $q=(x,y)$ , and the goal is to report the sum of the weights assigned to the points dominated by  $q$ , where a point  $(x',y')$  is dominated by  $q$  if  $x' \leq x$  and  $y' \leq y$ . In addition to being the highest cell probe lower bound to date, our lower bound is also tight for data structures with update time  $t_u = \Omega(\lg^{2+\epsilon} n)$ , where  $\epsilon > 0$  is an arbitrarily small constant.

This work will be presented at STOC'12.