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**MADALGO seminar by Martin Olsen, Aarhus University**

**Maximizing PageRank with new Backlinks**

**Abstract:**

The founders of Google introduced the PageRank algorithm that computes an estimate of the popularity of each web page based solely on the link structure of the web graph - these estimates are the so called PageRank values. A page will achieve one of the top spots of search results if it has a high PageRank value and it matches the search criteria for the actual Google search.

For a given node  $t$  in a directed graph  $G(V, E)$  and a positive integer  $k$  we study the problem of computing a set of  $k$  new links pointing to  $t$  - so called backlinks to  $t$  - producing the maximum increase in the PageRank value of  $t$ . This problem is known as "Link Building" in the www context. We present a theorem showing how the topology of the graph comes in to play when evaluating potential new backlinks. Based on the theorem we show that no FPTAS exists for Link Building under the assumption  $NP \neq P$  and we also show that Link Building is  $W[1]$ -hard - strongly suggesting that Link Building is not fixed parameter tractable. We prove these results by reduction from the independent set problem on regular graphs. Finally, we use the theorem to characterize sets of backlinks producing a significant increase in the PageRank value of  $t$ .