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MADALGO seminars by Gerth Stølting Brodal, Aarhus University

Integer Representations towards Efficient Counting in the Bit Probe Model

Abstract:
We consider the problem of representing numbers in close to optimal space and supporting increment, decrement, addition and subtraction operations efficiently. We study the problem in the bit probe model and analyse the number of bits read and written to perform the operations, both in the worst-case and in the average-case. A counter is *space-optimal* if it represents any number in the range $[0,...,2^n-1]$ using exactly n bits. We provide a *space-optimal counter* which supports increment and decrement operations by reading at most n-1 bits and writing at most 3 bits in the worst-case. To the best of our knowledge, this is the first such representation which supports these operations by always reading strictly less than n bits. For *redundant counters* where we only need to represent numbers in the range $[0,...,L]$ for some integer $L < 2^n -1$ using n bits, we define the efficiency of the counter as the ratio between $L+1$ and 2^n . We present various representations that achieve different trade-offs between the read and write complexities and the efficiency. We also give another representation of integers that uses $n + O(\log n)$ bits to represent integers in the range $[0,...,2^n-1]$ that supports efficient addition and subtraction operations, improving the space complexity of an earlier representation by Munro and Rahman [Algorithmica, 2010].

Joint work with Mark Greve, Vineet Pandey, and S. Srinivasa Rao.