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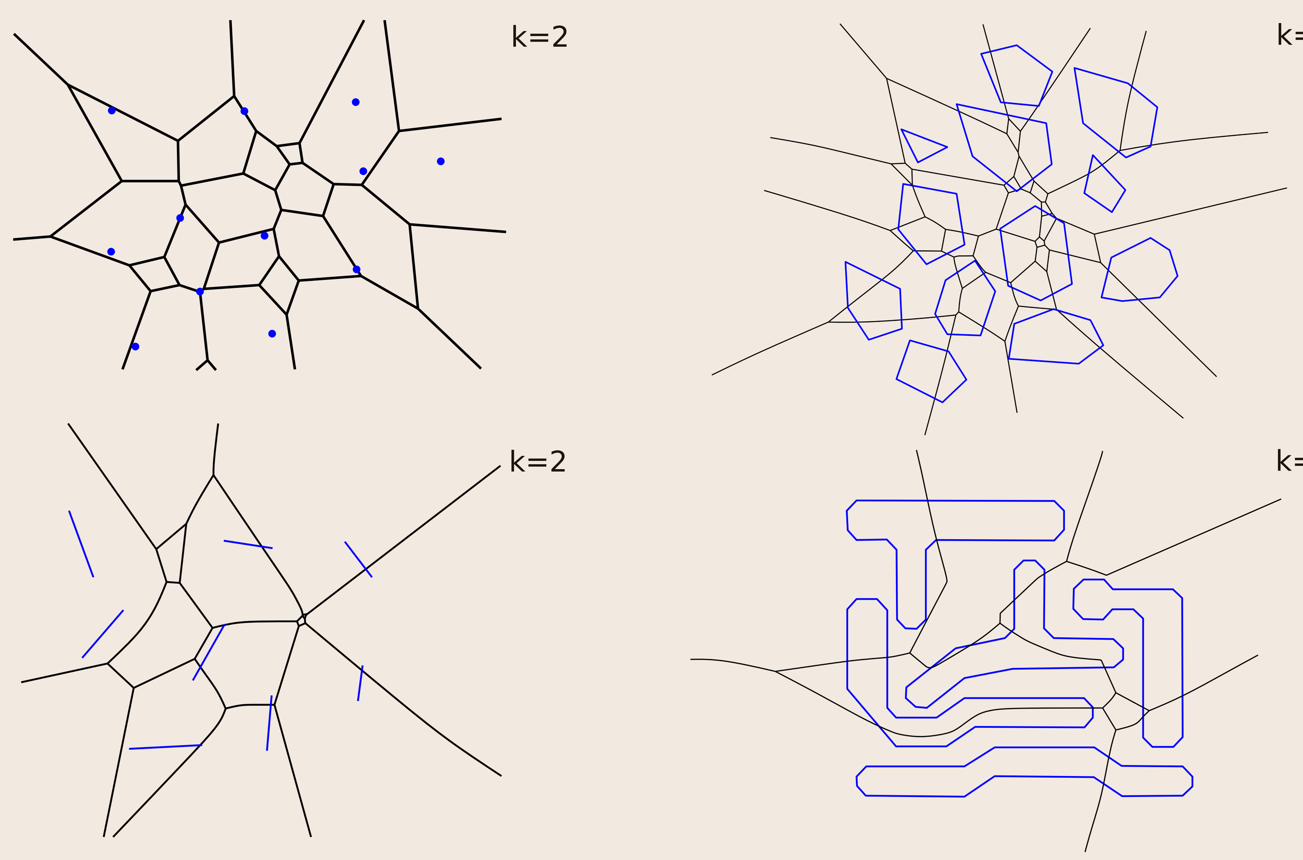
Constructing Higher-Order Voronoi Diagrams of Polygonal Objects

Definition

Order-k Voronoi diagram is a partitioning of the plane into regions such that every point within a fixed region has the same k-closest sites.



Higher-order Voronoi diagrams is a natural generalization of ordinary Voronoi diagrams. However, higher-order Voronoi diagrams were studied only for the case of points. In our research we want to study them for the case of polygonal objects like: line segments, convex polygons and simple polygons. We have studied structural complexity for some of the cases and developed construction algorithms based on the iterative and sweepline paradigms for the case of line segments.
Currently we are developing randomized algorithms and studying the structural complexity for the case of simple polygons.

Previous Results on the Structural Complexity

Points: $O(k(n-k))$ [1]

Our Results on the Structural Complexity

Line Segments [2]:

- disjoint or abutting in L_p metric: $O(k(n-k))$
- intersecting in L_p metric: $O(k(n-k)+I),\ k<n/2$; $O(k(n-k)),\ k\geq n/2$
- disjoint in L_1 or L_∞ metrics: $O\left(\min\left\{k(n-k),(n-k)^2\right\}\right)$

Convex polygons [3]:

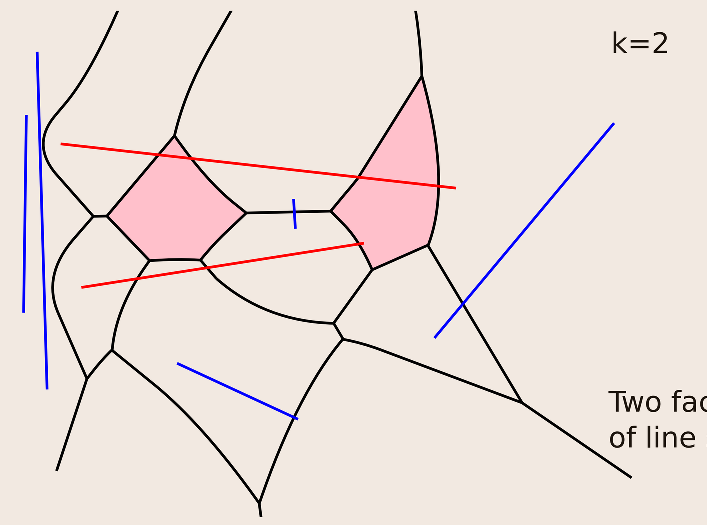
- disjoint and of constant size: $O(k(n-k))$

Abstract Voronoi diagrams [3]: $O(k(n-k))$

* where $1\leq p\leq\infty$

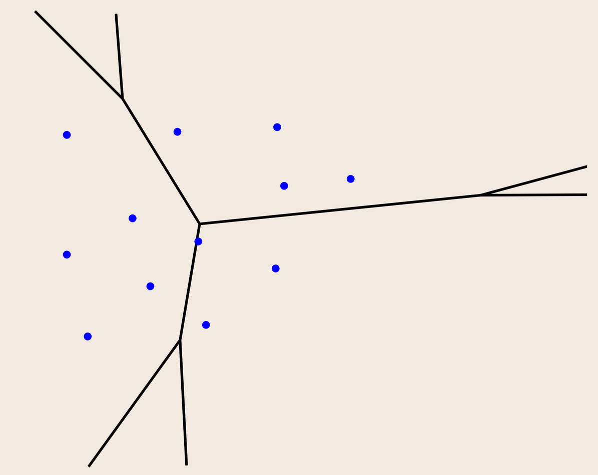
Differences with Points

In case of higher-order Voronoi diagrams of polygonal objects a single order-k Voronoi region may be **disconnected** into Ω(n) faces, for k>1. This is not the case for point sites.

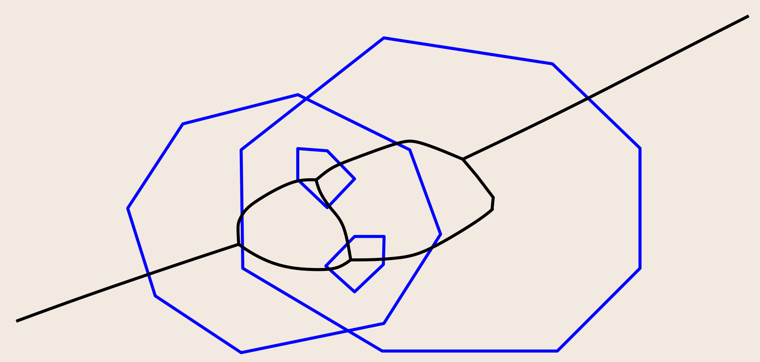


Two faces induced by the same pair of line segments.

In general the farthest Voronoi diagram **may not be a tree** in case of convex polygons or simple polygons. In case of point sites the farthest Voronoi diagram is always a tree.



For points the farthest Voronoi diagram is always a tree.



For convex polygons and simple polygons the farthest Voronoi diagram may not be a tree.

Polygonal objects may intersect producing bisectors that are **not Jordan curves**. In case of point sites bisector is always a line.

Even in the case of disjoint simple polygons sites may enclose other sites in their "pockets", producing closed bisectors.

Previous Results on the Algorithms

For points:

Construction Time	Reference	Construction Time	Reference
$O(k^2 n\log n)$	Lee'82	$O(k^2 n\log n + nk\log^3 n)$	Aurenhammer &Schwarzkopf'92
$O(n^2 + b\log^2 n)$	Chazelle&Edelsbrunenr'87	$O(k^3 n + n\log n)$	Boissonnat et al.'93
$O(n^{1+\epsilon}k)$	Clarkson'87	$O(n\log^3 n + nk\log n)$	Agarwal et al.'98
$O(k^2 n\log n)$	Aurenhammer'90	$O(n\log n + nk\log n)$	Chan'98
$O(k^2 n + n\log n)$	Mulmuley'91	$O\left(n\log n + nk2^{c\log^* k}\right)$	Ramos'99

Our Results on the Algorithms

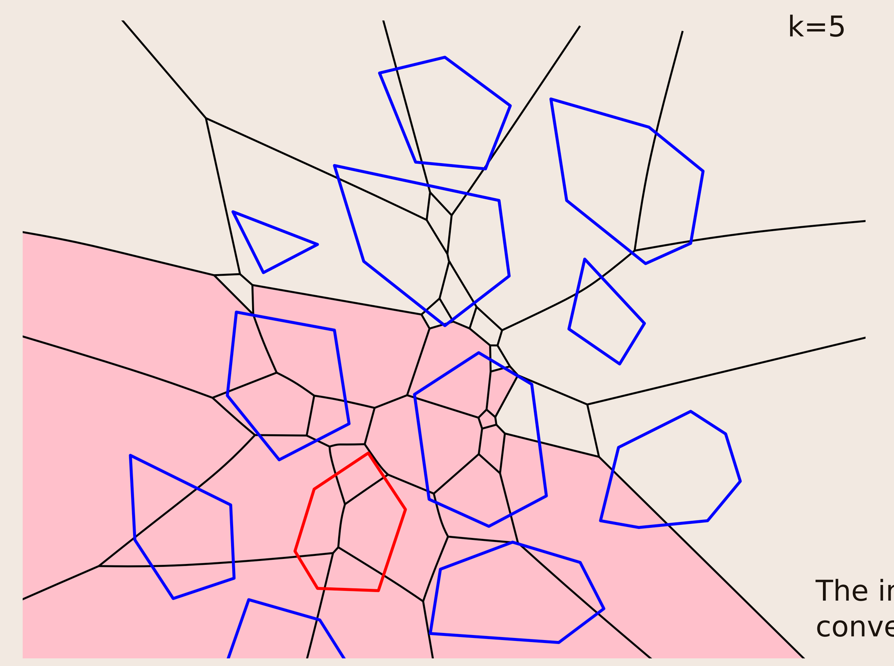
Iterative approach is based on a simple paradigm: construct the order-(i+1) Voronoi diargam out of the order-i Voronoi diagram by repeatedly partitioning the faces of the order-i Voronoi diagram. This approach can be easily generalized to other geometric objects. The running time is:

For disjoint or abutting line segments: $O(k^2 n + n\log n)$ expected

Sweepline approach can be applied to construct higher-order Voronoi diagrams by generalizing Fortune's algorithm for k≥1. So far we have developed it for the case of points, disjoint line segments and line segments forming PSLG.

For points, disjoint or abutting line segments: $O(k^2 n\log n)$ [4]

Randomized construction algorithms is the main topic of our research. Currently we are focusing on a simple version of a randomized algorithm. The main idea is to construct the order-k Voronoi diagram with influence regions. The influence region $V_k(s)$ is the union of regions to which site s constributes. For disjoint line segments and convex polygons $V_k(s)$ is a connected set, and the structure of the boundary is very similar to the structure of k-level of x-monotone curves. We are planning to apply already known construction algorithms for k-levels, like Har-Peled's randomized algorithm [5] to construct all the influnece regions of the order-k Voronoi diagram.



The influence region of the convex polygon.

As our future research we are considering probabilistic divide and conquer method [6]. The idea of this method is to draw a random sample of sites that efficiently "brackets" the space. Using the random sample we can do a recursive call on the smaller subsets of sites. The key point is to define a "bracketing" of the space in such a way that we can find a "good" random sample within expected constant number of trials.

[1] D.T. Lee. On k-Nearest Neighbor Voronoi Diagrams in the Plane. IEEE Trans. Computers 31(6): 478-487 (1982)
[2] E. Papadopoulou, M. Zavershynskyi. On the higher order Voronoi diagram of line segments. ISAAC, 177–186, 2012.
[3] C. Bohler, R. Klein, C.-H. Liu, E. Papadopoulou, P. Cheilaris, M. Zavershynskyi. On the complexity of higher order abstract voronoi diagrams. ICALP volume 7965, pages 208–219, 2013.
[4] M. Zavershynskyi, E. Papadopoulou. A Sweepline Algorithm for Higher-Order Voronoi Diagrams. ISVD (2013)
[5] Sariel Har-Peled. Taking a Walk in a Planar Arrangement. SIAM J. Comput 30(4): 1341-1367 (2000)
[6] K.L. Clarkson. New applications of random sampling in computational geometry. Discrete and Computational Geometry. 2:195-222 (1987)